IIT-JAM 2020

SECTION – A

MULTIPLE CHOICE QUESTIONS (MCQ)

Q1 – Q10 carry one mark each.

Q1. Which one of the following functions has a discontinuity in the second derivative at $x=0$, where x is a real variable?

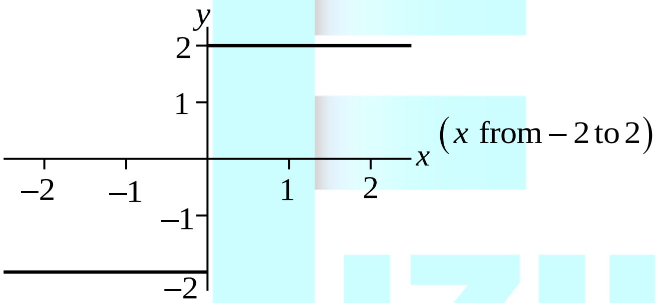
(a) $f(x)=|x|^3$ (b) $f(x)=x|x|$ (c) $f(x)=\cos(|x|)$ (d) $f(x)=|x|^2$

Ans. : (b)

Solution: $\frac{d^2}{dx^2}(x|x|)=\frac{2x}{|x|}$

Assuming a function to be real.

Plots:



Q2. A collimated beam of laser light of wavelength $514\,nm$ is normally incident on a smooth glass slab placed in air. Given the refractive indices of glass and air are 1.47 and 1.0, respectively, the percentage of light intensity reflected back is

(a) 0 (b) 4.0 (c) 3.6 (d) 4.2

Ans. : (c)

Solution: $R=\left(\frac{n_1-n_2}{n_1+n_2}\right)^2=\left(\frac{1-1.47}{1+1.47}\right)=0.036=0.036\times 100=3.6\%$

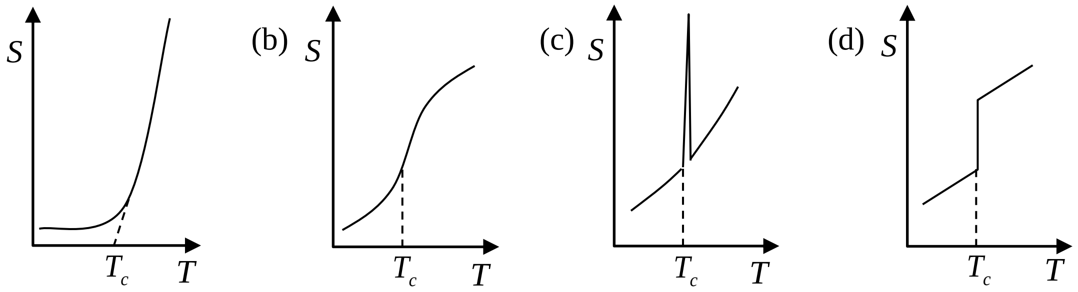
Q3. Two stationary point particles with equal and opposite charges are at some fixed distance from each other. The points having zero electric potential lie on:

(a) A sphere (b) A plane (c) A cylinder (d) Two parallel planes

Ans. : (b)

Solution: All points on the plane are at equal distance from charges, so potential will be zero.

Q4. For a system undergoing a first order phase transition at a temperature T_c, which one of the following graphs best describes the variation of entropy (S) as a function of temperature (T)?



Ans. : (d)

Solution: $S=-\left(\frac{\partial g}{\partial T}\right)_P$ specific entropy. In first order Phase transition first order derivative of specific Gibbs free energy is discontinuous that is specific entropy.

Q5. In a photoelectric effect experiment, a monochromatic light source emitting photon with energy greater than the work function of the metal under test is used. If the power of the light source is doubled, which one of the following statements is correct?

(a) The number of emitted photoelectrons remains the same

(b) The stopping potential remains the same

(c) The number of emitted photoelectrons decreases

(d) The stopping potential doubles

Ans.: (b)

Solution: Stopping potential is not defined at power of light source

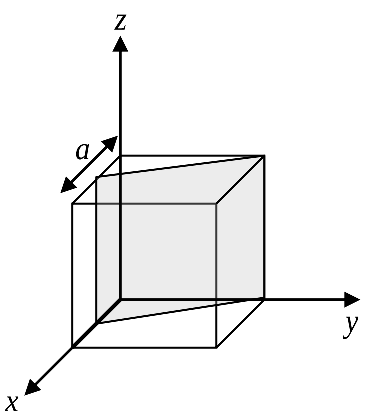
Q6. The figure below shows a cubic unit cell with lattice constant a. The shaded crystallographic plane intersects the x-axis at $0.5a$. The Miller indices of the shaded plane are

(a) (210)

(b) $(\bar{2}10)$

(c) $(1\bar{1}0)$

(d) $(10\bar{2})$



Ans. : (a)

Solution: (i) Intercepts along x, y, z - axis

$$x=\frac{a}{2},\ y=a,\ z=\infty$$

(ii) Divide by lattice parameters

$\frac{a/2}{a}, \frac{a}{a}, \frac{\infty}{a} \Rightarrow \frac{1}{2}, 1, \infty$

(iii) Take reciprocal, $2,1,0$

$\therefore$ The Miller indices is $(2\;1\;0)$

Q7. For a particle moving in a central potential, which one of the following statements is correct?

(a) The motion is restricted to a plane due to the conservation of angular momentum

(b) The motion is restricted to a plane due to the conservation of energy only

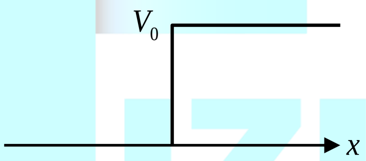
(c) The motion is restricted to a plane due to the conservation of linear momentum

(d) The motion is not restricted to a plane

Ans. : (a)

Solution: For central force problem angular momentum $\vec{J}$ is conserved and $\vec{r}.\vec{J}=0$ which ensure that motion of particle is confined in plane

Q8. Consider the motion of a quantum particle of mass m and energy E under the influence of a step potential of height V_0. If R denotes the reflection coefficient, which one of the following statements is true?



(a) If $E=\frac{4}{3}V_0,\ R=1$

(b) $E=\frac{4}{3}V_0,\ R=0$

(c) $E=\frac{1}{2}V_0,\ R=1$

(d) $E=\frac{1}{2}V_0,\ R=0.5$

Ans. : (c)

Solution: $E=\frac{V_0}{2} \quad \Rightarrow E<V_0$, so $R=1$

Therefore, option (c) is correct

Q9. The Boolean function $\overline{PQ}\left(\bar{P}+Q\right)\left(Q+\bar{Q}\right)$ is equivalent to:

(a) P

(b) $\bar{P}$

(c) $\bar{P}Q$

(d) $P\bar{Q}$

Ans. : (b)

Solution: $Y=\overline{PQ}\left(\bar{P}+Q\right)\left(Q+\bar{Q}\right) \Rightarrow Y=\left(\bar{P}+\bar{Q}\right)\left(\bar{P}+Q\right).1=\bar{P}+\bar{P}Q+\bar{P}\bar{Q}$

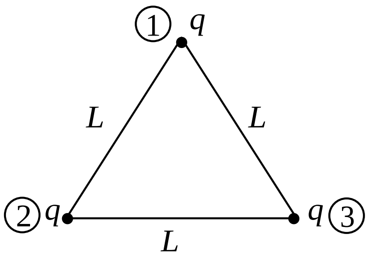
$\Rightarrow Y=\bar{P}(1+Q)+\bar{P}\bar{Q}=\bar{P}\left(1+\bar{Q}\right)=\bar{P}$

Q10. Three point charges each carrying a charge q are placed on the vertices of an equilateral triangle of side L. The electrostatic potential energy of the configuration is:

(a) $\frac{1}{4\pi\varepsilon_0}\frac{q^2}{L}$ (b) $\frac{2}{4\pi\varepsilon_0}\frac{q^2}{L}$ (c) $\frac{3}{4\pi\varepsilon_0}\frac{q^2}{L}$ (d) $\frac{1}{\pi\varepsilon_0}\frac{q^2}{L}$

Ans. 10: (c)

Solution: Work done in placing charges



$W_1 = 0$, $W_2 = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{L}$

and $W_3 = q\frac{1}{4\pi\varepsilon_0}\left(\frac{q}{L}+\frac{q}{L}\right) = \frac{1}{4\pi\varepsilon_0}\frac{2q^2}{L}$

Also Total work done $= W_1 + W_2 + W_3 = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{L}(1+2) = \frac{3}{4\pi\varepsilon_0}\frac{q^2}{L}$

Q11 - Q30 carry two marks each.

Q11. Which of the following statements is correct?

Given $\binom{n}{m}=\frac{n!}{m!(n-m)!}$ is the binomial coefficient

(a) $\cos n\theta=\cos^n\theta-\binom{n}{2}\cos^{n-2}\theta\sin^2\theta+\binom{n}{4}\cos^{n-4}\theta\sin^4\theta-...$

(b) $\sin n\theta=\binom{n}{1}\cos^{n-1}\theta\sin\theta+\binom{n}{3}\cos^{n-3}\theta\sin^3\theta+...$

(c) $\cos n\theta=\cos^n\theta+\binom{n}{2}\cos^{n-2}\theta\sin^2\theta+\binom{n}{4}\cos^{n-4}\theta\sin^4\theta+....$

(d) $\sin n\theta=\cos^n\theta-\binom{n}{2}\cos^{n-2}\theta\sin^2\theta+\binom{n}{4}\cos^{n-4}\theta\sin^4\theta-....$

Ans. : (a)

Solution: Let's invoke De Moivre's theorem here. We have

$$\cos(n\theta)+i\sin(n\theta)=(\cos(\theta)+i\sin(\theta))^n$$

Since n is a positive integer, the binomial theorem holds,

$$(\cos(\theta)+i\sin(\theta))^n$$

Hence, by expanding, we have

$$(\cos(\theta)+i\sin(\theta))^n=\cos^n(\theta)+n\cos^{n-1}(\theta)\cdot i\sin(\theta)+\frac{n(n-1)}{1\cdot 2}\cos^{n-2}(\theta)\cdot i^2\sin^2(\theta)+....$$

Since $i^2=-1,\ i^3=-i,\ i^4=1,\ i^5=i,....$, we have

$$(\cos(\theta)+i\sin(\theta))^n=\left[\cos^n(\theta)-\frac{n(n-1)}{1\cdot 2}\cos^{n-2}(\theta)\cdot\sin^2(\theta)\right.$$

$$\left.+\frac{n(n-1)(n-2)(n-3)}{1\cdot 2\cdot 3\cdot 4}\cos^{n-4}(\theta)\cdot\sin^4(\theta)+....\right]$$

$$+i\left[n\cos^{n-1}(\theta)\cdot\sin(\theta)-\frac{n(n-1)(n-2)(n-3)}{1\cdot 2\cdot 3}\cos^{n-3}(\theta)\cdot\sin^3(\theta)+....\right]$$

By equating the real and imaginary parts, we obtain

$$\cos(n\theta)=\cos^n(\theta)-\frac{n(n-1)}{1\cdot 2}\cos^{n-2}(\theta)\cdot\sin^2(\theta)$$

$$+\frac{n(n-1)(n-2)(n-3)}{1\cdot 2\cdot 3\cdot 4}\cos^{n-4}(\theta)\cdot\sin^4(\theta)+....$$

Also, $\sin(n\theta) = n\cos^{n-1}(\theta)\cdot\sin(\theta) - \frac{n(n-1)(n-2)}{1\cdot 2\cdot 3}\cos^{n-3}(\theta)\cdot\sin^3(\theta) + ...$

Q12. The volume integral $\int_v e^{-\left(\frac{r}{R}\right)^2}\vec{\nabla}\cdot\left(\frac{\hat{r}}{r^2}\right)d^3r$, where V is the volume of a sphere of radius R centered at the origin, is equal to

(a) 4π (b) 0 (c) $\frac{4}{3}\pi R^3$ (d) 1

Ans. 12: (a)

Solution: $J = \int_v e^{-\left(\frac{r}{R}\right)^2}\vec{\nabla}\cdot\left(\frac{\hat{r}}{r^2}\right)d^3r = \int_v e^{-\left(\frac{r}{R}\right)^2}\times 4\pi\delta^3(r)d^3r = 4\pi\left[e^{-\left(\frac{0}{R}\right)^2}\right] = 4\pi$

Q13. $\lim_{x\to 0+} x^x$ is equal to

(a) 0 (b) ∞ (c) e (d) 1

Ans. : (d)

Solution: $y = x^x \Rightarrow \ln y = x\ln x = \frac{\ln x}{1/x}$

$\lim_{x\to 0}\ln y = \frac{1/x}{-1/x^2} = -x = 0$, $\ln y = 0$, So, $y = 1$ is the answer.

$\Rightarrow y = e^0 = 1$

Q14. A wheel is rotating at a frequency $f_0\,Hz$ about a fixed vertical axis. The wheel stops in t_0 seconds, with constant angular deceleration. The number of turns covered by the wheel before it comes to rest is given by

(a) f_0t_0 (b) $2f_0t_0$ (c) $\frac{f_0t_0}{2}$ (d) $\frac{f_0t_0}{\sqrt{2}}$

Ans. : (c)

Solution: $\omega_0 = 2\pi f_0$, $\omega = \omega_0 - \alpha t$

$\alpha = \frac{\omega_0}{t_0} \Rightarrow \alpha = \frac{2\pi f_0}{t_0}$

$\theta = \omega_0 t - \frac{1}{2}\alpha t^2 = \omega_0 t_0 - \frac{1}{2}\frac{\omega_0}{t_0}t_0^2 = \frac{\omega_0 t_0}{2} = \frac{2\pi f_0 t_0}{2}$

Number of turns $= \frac{\theta}{2\pi} = \frac{f_0t_0}{2}$. So the option (c) is correct.

Q15. Two objects of masses m and $2m$ are moving at speeds of v and $v/2$, respectively. After undergoing a completely inelastic collision, they move together with a speed of $v/3$. The angle between the initial velocity of the two objects is

(a) 60^0 (b) 120^0 (c) 45^0 (d) 90^0

Ans. : (b)

Solution: As $\Delta p_x = 0,\ \Delta p_y = 0$

For x direction,

$$\Rightarrow 2m \times \frac{v}{2}\frac{\cos\theta}{2} + mv\frac{\cos\theta}{2} = 3m \times \frac{v}{3}$$

$$\Rightarrow 2mv\frac{\cos\theta}{2} = mv \Rightarrow \frac{\cos\theta}{2} = \frac{1}{3} = \cos 60^0$$

$$\frac{\theta}{2} = 60^0 \Rightarrow \theta = 120^0$$

Q16. Two planets P_1 and P_2 having masses M_1 and M_2 revolve around the Sun in elliptical orbits, with time periods T_1 and T_2 respectively. The minimum and maximum distances of planet P_1 from the Sun are R and $3R$ respectively, whereas for planet P_2 these are $2R$ and $4R$, respectively, where R is a constant. Assuming M_1 and M_2 are much smaller than the mass of the Sun, the magnitude of $\frac{T_2}{T_1}$ is

(a) $\frac{2}{3}\sqrt{\frac{2M_1}{3M_2}}$ (b) $\frac{3}{2}\sqrt{\frac{3M_2}{2M_1}}$ (c) $\frac{3}{2}\sqrt{\frac{3}{2}}$ (d) $\frac{2}{3}\sqrt{\frac{2}{3}}$

Ans. : (c)

Solution: $T^2 \propto a^3$ For P_1, $2a_1 = 4R \Rightarrow a_1 = 2R$

For P_2, $2a_2 = 6R \Rightarrow a_2 = 3R$

$$T^2 \propto \frac{4\pi^2 a^3}{GM_s}$$

$$\frac{T_2}{T_1} = \left(\frac{a_2}{a_1}\right)^{3/2} = \left(\frac{3R}{2R}\right)^{3/2} = \left(\frac{3}{2}\right)^{3/2} = \frac{3}{2}\frac{\sqrt{3}}{2}$$

option (c) is correct

Q17. The intensity of the primary maximum in a two-slit interference pattern is given by I_2 and the intensity of the primary maximum in a three-slit interference pattern is given by I_3. Assuming the far-field approximation, same slit parameters and intensity of the incident light in both the cases, I_2 and I_3 are related as

(a) $I_2 = \frac{3}{2} I_3$ (b) $I_2 = \frac{9}{4} I_3$ (c) $I_2 = \frac{2}{3} I_3$ (d) $I_2 = \frac{4}{9} I_3$

Ans. : (d)

Solution: For two slits, $I_2 = (a+a)^2 = 4a^2$

For three slits, $I_3 = (a+a+a)^2 = 9a^2$

$\therefore \frac{I_2}{I_3} = \frac{4}{9} \Rightarrow I_2 = \frac{4}{9} I_3$

Q18. A short rod of length L and negligible diameter lies along the optical axis of concave mirror at a distance of $3m$. The focal length of the mirror is $1m$ and $L << 1m$. If L' is the length of image of the object in the mirror, then

(a) $\frac{L'}{L} = 4$ (b) $\frac{L'}{L} = 2$ (c) $\frac{L'}{L} = \frac{1}{16}$ (d) $\frac{L'}{L} = \frac{1}{4}$

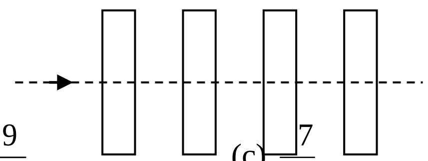
Ans. : (d)

Solution: For mirror $\frac{1}{u} + \frac{1}{v} = \frac{1}{f} \Rightarrow -\frac{du}{u^2} - \frac{dv}{v^2} = 0 \Rightarrow \frac{dv}{du} = -\frac{v^2}{u^2} \to \frac{|dv|}{|du|} = m^2$

$\frac{L'}{L} = m^2$

$m = \frac{v}{u} = \frac{f}{u-f} = \frac{(-1)}{(-3)-(-1)} = \frac{1}{2}$ $\because u = -3m,\ f = -1m$ $\therefore \frac{L'}{L} = \frac{1}{4}$

Q19. A beam of unpolarized light of intensity I_0 falls on a system of four identical linear polarizers placed in a line as shown in figure. The transmission axes of any two successive polarizers make an angle of 30^0 with each other. If the transmitted light has intensity I, the ratio $\frac{I}{I_0}$ is



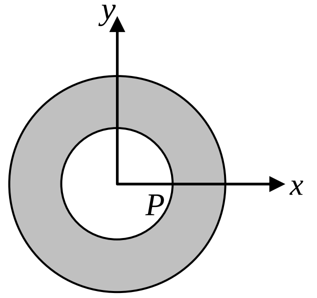
(a) $\frac{81}{256}$ (b) $\frac{9}{16}$ (c) $\frac{7}{64}$ (d) $\frac{27}{128}$

Ans. : (d)

Solution: $I_1 = \frac{I_0}{2}$, $I_2 = \frac{I_0}{2}\cos^2 30^0$, $I_3 = I_2 \cos^2 30 = \frac{I_0}{2}\cos^4 30^0$

$$I_4 = I_3 \cos^2 30^0 = \frac{I_0}{2}\cos^6 30^0 = \frac{I_0}{2}\left(\frac{\sqrt{3}}{2}\right)^6 = \frac{27}{128} I_0$$

Q20. Consider an annular region in free space containing a uniform magnetic field in the z-direction, schematically represented by the shaded region in the figure. A particle having charge Q and mass M starts off from point $P(a,0,0)$ in the $+x$- direction with constant speed v. If the radii of inner and outer circles are a and b, respectively, the minimum magnetic field required so that the particle returns to the inner circle is

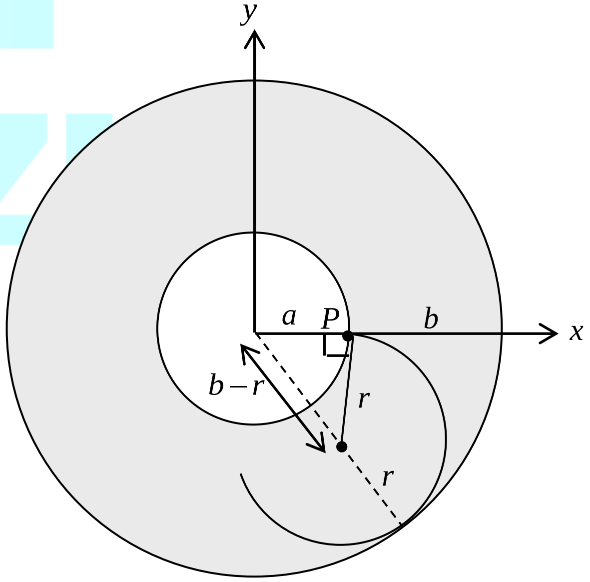


(a) $\frac{Mv}{Q}\left(\frac{b^2-a^2}{b}\right)^{-1}$ (b) $\frac{Mv}{Q}\left(\frac{b^2-a^2}{2b}\right)^{-1}$

(c) $\frac{Mv}{Q}\left(\frac{b^2-a^2}{3b}\right)^{-1}$ (d) $\frac{Mv}{Q}\left(\frac{b^2-a^2}{4b}\right)^{-1}$

Ans. 20: (b)

Solution: In the right angle triangle shown in figure



$$(b-r)^2 = a^2 + r^2$$

$$\Rightarrow b^2 + r^2 - 2br = a^2 + r^2$$

$$\Rightarrow r = \frac{b^2-a^2}{2b}$$

Radius of the circular path is

$$r = \frac{mv}{QB}$$

$$\Rightarrow B = \frac{mv}{Qr} = \frac{mv}{Q}\left(\frac{b^2-a^2}{2b}\right)^{-1}$$

Q21. A thin conducting square loop of side L is placed in the first quadrant of the xy- plane with one of the vertices at the origin. If a changing magnetic field $\vec{B}(t) = \beta_0\left(5zyt\,\hat{x} + zxt\,\hat{y} + 3y^2t\,\hat{z}\right)$ is applied, where β_0 is a constant, then the magnitude of the induced electromotive force in the loop is

(a) $4\beta_0 L^4$ (b) $3\beta_0 L^4$ (c) $2\beta_0 L^4$ (d) $\beta_0 L^4$

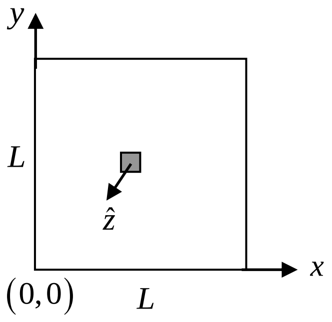
Ans. 21: (d)

Solution: Area element $d\vec{a} = dxdy\hat{z} \Rightarrow \vec{B}.d\vec{a} = 3\beta_0 y^2 t dxdy$

Magnetic flux $\phi = \int_S \vec{B}.d\vec{a} = \int_S 3\beta_0 y^2 t dx dy$

$$\Rightarrow \phi = 3\beta_0 t \int_0^L dx \int_0^L y^2 dy = 3\beta_0 t \times L \times \frac{L^3}{3} = \beta_0 L^4 t$$

Thus magnitude of e.m.f $|\varepsilon| = \frac{d\phi}{dt} = \beta_0 L^4$



Q22. In which one of the following limits the Fermi-Dirac distribution $n_F(\varepsilon, T) = \left(e^{\frac{\varepsilon-\mu}{k_B T}} + 1\right)^{-1}$ and Bose-Einstein distribution $n_B(\varepsilon, T) = \left(e^{\frac{\varepsilon-\mu}{k_B T}} - 1\right)^{-1}$ reduce to Maxwell-Boltzmann distribution?

(Here ε is the energy of the state, μ is the chemical potential, k_B is the Boltzmann constant and T is the temperature)

(a) $\mu = 0$ (b) $(\varepsilon - \mu) \square\ k_B T$

(c) $(\varepsilon - \mu) \square\ k_B T$ (d) $\mu \square\ k_B T$

Ans. : (c)

Solution: $n_F(\varepsilon, T) = \frac{1}{1 + e^{\frac{(\varepsilon-\mu)}{k_B T}}}$, $n_B(\varepsilon, T) = \frac{1}{e^{\frac{(\varepsilon-\mu)}{k_B T}} - 1}$

Let $\frac{(\varepsilon - \mu)}{k_B T} = x$, If $\frac{(\varepsilon - \mu)}{k_B T} \square\ 1 \Rightarrow (\varepsilon - \mu) \square\ k_B T$

If $x \square\ 1 \Rightarrow e^x \square\ 1$, then $n_F(\varepsilon, T) = e^{-x}$, $n_B(\varepsilon, T) = e^{-x}$ which is M.B.

Q23. Consider N classical particles at temperature T, each of which can have two possible energies 0 and ε. The number of particles in the lower energy level (N_0) and higher energy level (N_ε) levels are related by

(k_B is the Boltzmann constant)

(a) $\frac{N_0}{N_\varepsilon} = e^{\frac{-\varepsilon}{k_B T}}$ (b) $\frac{N_0}{N_\varepsilon} = e^{\frac{\varepsilon}{k_B T}}$ (c) $\frac{N_0}{N_\varepsilon} = 1 + e^{\frac{\varepsilon}{k_B T}}$ (d) $\frac{N_0}{N_\varepsilon} = 1 - e^{\frac{-\varepsilon}{k_B T}}$

Ans. : (b)

Solution: $\because N_E = N_0 e^{-\beta E} \Rightarrow \frac{N_E}{N_0} = e^{-\beta E} \Rightarrow \frac{N_0}{N_E} = e^{\beta E}$

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Q24. The root mean square (rms) speeds of Hydrogen atoms at $500\,K$, V_H and Helium atoms at $2000\,K, V_{He}$ are related as

(a) $V_H > V_{He}$ (b) $V_H < V_{He}$ (c) $V_H = V_{He}$ (d) V_H □ V_{He}

Ans. : (c)

Solution: $V_{rms} = \sqrt{\dfrac{3k_BT}{m}}, V_H = \sqrt{\dfrac{3RT_H}{M_H}}$ and $V_{He} = \sqrt{\dfrac{3RT_{He}}{M_{He}}}$

$$\frac{V_H}{V_{He}} = \sqrt{\frac{T_H}{M_H} \times \frac{M_{He}}{T_{He}}} = \sqrt{\frac{500}{2000} \times \frac{4}{2}} \quad \Rightarrow \frac{V_H}{V_{He}} = 0.7070.$$

Therefore $V_H < V_{He}$

Q25. The normalized ground-state wave function of a one-dimensional quantum harmonic oscillator with force constant K and mass m is $\psi_0(x) = \left(\dfrac{\alpha}{\pi}\right)^{1/4} e^{-\alpha x^2/2}$, where $\alpha = m\omega_0/\hbar$ and $\omega_0^2 = K/m$. Which one of the following is the probability of finding the particle outside the classically allowed region? (The classically allowed region is where the total energy is greater than the potential energy)

(a) $\dfrac{2}{\sqrt{\pi}}\displaystyle\int_1^\infty y^2 e^{-y^2} dy$ (b) $\dfrac{2}{\sqrt{\pi}}\displaystyle\int_1^\infty e^{-y^2} dy$

(c) 0.5 (d) 0

Ans. : (b)

Solution: Turning point is

$$\alpha_1 = -\sqrt{\frac{2E}{m\omega_0^2}} = -\sqrt{\frac{\hbar}{m\omega_0}} \text{ and } \alpha_2 = +\sqrt{\frac{\hbar}{m\omega_0}}$$

Probability to find the particle in classical forbidden region

$$2\int_\alpha^\infty |\psi_0|^2 dx = 2\int_{\sqrt{\frac{\hbar}{m\omega_0}}}^\infty \left(\frac{\alpha}{\pi}\right)^{1/2} e^{-\alpha x^2} dx = 2\times\left(\frac{m\omega_0}{\pi\hbar}\right)^{1/2} \int_{\sqrt{\frac{\hbar}{m\omega_0}}}^\infty e^{-\frac{m\omega_0 x^2}{\hbar}} dx$$

Let $\sqrt{\dfrac{m\omega_0}{\hbar}}x = y \Rightarrow dx = \sqrt{\dfrac{\hbar}{m\omega_0}}dy$

If $x = \sqrt{\dfrac{\hbar}{m\omega_0}} \Rightarrow y = 1$ and if $x = \infty \Rightarrow y = \infty$

Thus $2\times\left(\dfrac{m\omega_0}{\pi\hbar}\right)^{1/2}\sqrt{\dfrac{\hbar}{m\omega_0}}\displaystyle\int_1^\infty e^{-y^2} dy = \dfrac{2}{\sqrt{\pi}}\displaystyle\int_1^\infty e^{-y^2} dy$

Therefore, option (b) is correct.

Q26. A linear operator $\hat{O}$ acts on two orthonormal states of a system ψ_1 and ψ_2 as per following: $\hat{O}\psi_1 = \psi_2,\ \hat{O}\psi_2 = \frac{1}{\sqrt{2}}(\psi_1 + \psi_2)$. The system is in a superposed state defined by $\psi = \frac{1}{\sqrt{2}}\psi_1 + \frac{i}{\sqrt{2}}\psi_2$. The expectation value of $\hat{O}$ in the state ψ is

(a) $\frac{1}{2\sqrt{2}}\left(1 + i\left(\sqrt{2}+1\right)\right)$

(b) $\frac{1}{2\sqrt{2}}\left(1 - i\left(\sqrt{2}+1\right)\right)$

(c) $\frac{1}{2\sqrt{2}}\left(1 + i\left(\sqrt{2}-1\right)\right)$

(d) $\frac{1}{2\sqrt{2}}\left(1 - i\left(\sqrt{2}-1\right)\right)$

Ans. : (d)

Solution: $\hat{O}|\psi_1\rangle = |\psi_2\rangle$

$$\hat{O}|\psi_2\rangle = \frac{1}{\sqrt{2}}|\psi_1\rangle + |\psi_2\rangle \qquad |\psi\rangle = \frac{1}{\sqrt{2}}(|\psi_1\rangle) + \frac{i}{\sqrt{2}}(|\psi_2\rangle)$$

$$\langle\psi| = \frac{1}{\sqrt{2}}\langle\psi_1| - \frac{i}{\sqrt{2}}\langle\psi_2|$$

$$\langle\hat{O}\rangle = \frac{\langle\psi|\hat{O}|\psi\rangle}{\langle\psi|\psi\rangle} \qquad \langle\psi|\psi\rangle = 1$$

$$\hat{O}|\psi\rangle = \frac{1}{\sqrt{2}}\hat{O}|\psi_1\rangle + \frac{i}{\sqrt{2}}\hat{O}|\psi_2\rangle = \frac{1}{\sqrt{2}}|\psi_2\rangle + \frac{i}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}|\psi_1\rangle + \frac{1}{\sqrt{2}}|\psi_2\rangle\right)$$

$$= \frac{i}{2}|\psi_1\rangle + \left(\frac{1}{\sqrt{2}} + \frac{i}{2}\right)|\psi_2\rangle = \langle\psi|\hat{O}|\psi\rangle = \frac{i}{2\sqrt{2}} - \frac{i}{\sqrt{2}}\left(\frac{1}{\sqrt{2}} + \frac{i}{2}\right)$$

$$= \frac{i}{2\sqrt{2}} - \frac{i}{2} + \frac{1}{2\sqrt{2}} = \frac{i}{2\sqrt{2}}\left(1 + \left(1-\sqrt{2}\right)i\right) = \frac{1}{2\sqrt{2}}\left(1 - i\left(\sqrt{2}-1\right)\right)$$

Q27. Consider a one-dimensional infinite potential well of width a. The system contains five non-interacting electrons, each of mass m, at temperature $T = 0K$. The energy of the highest occupied state is

(a) $\frac{25\pi^2\hbar^2}{2ma^2}$ (b) $\frac{10\pi^2\hbar^2}{2ma^2}$ (c) $\frac{5\pi^2\hbar^2}{2ma^2}$ (d) $\frac{9\pi^2\hbar^2}{2ma^2}$

Ans. : (d)

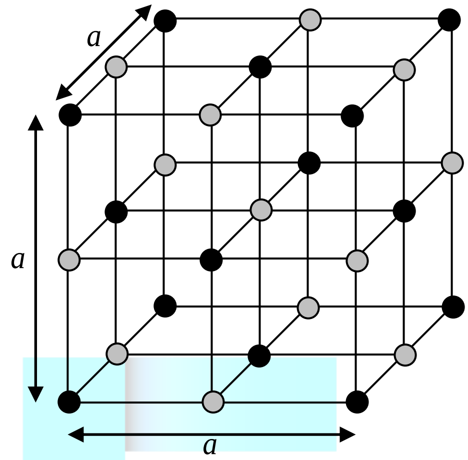
Solution: Ground state have two electrons so energy is $2E_0$.

First excited state has two electrons, $2\cdot(4E_0)$.

Second excited state has remaining 1 electron, $1\times 9E_0$ where $E_0 = \frac{\pi^2\hbar^2}{2ma^2}$

So, energy at highest occipital level is $9\times\frac{\pi^2\hbar^2}{2ma^2}$

Q28. Consider the crystal structure shown in the figure, where black and grey spheres represent atoms of two different elements and a denotes the lattice constant. The Bravais lattice for this structure is



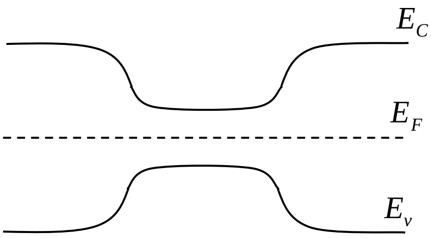
(a) Simple cubic
(b) Face-centered cubic
(c) Body-centered cubic
(d) Triclinic

Ans. : (b)

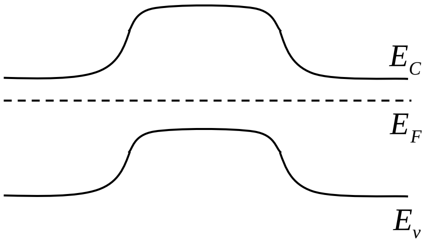
Solution: This is a structure of sodium-chloride whose lattice is Face Centred Cubic. Thus, the correct option is (b)

Q29. For unbiased Silicon *n-p-n* transistor in thermal equilibrium, which one of the following electronic energy band diagrams is correct? (E_c = conduction band minimum, E_v = valence band maximum, E_F =Fermi level).

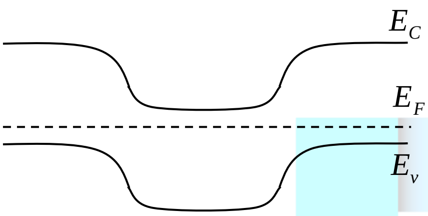
(a)



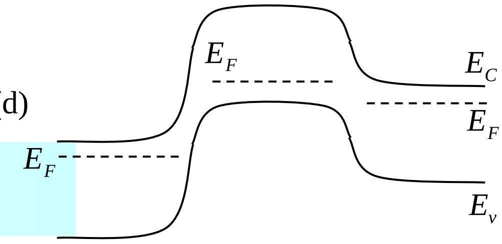
(b)



(c)



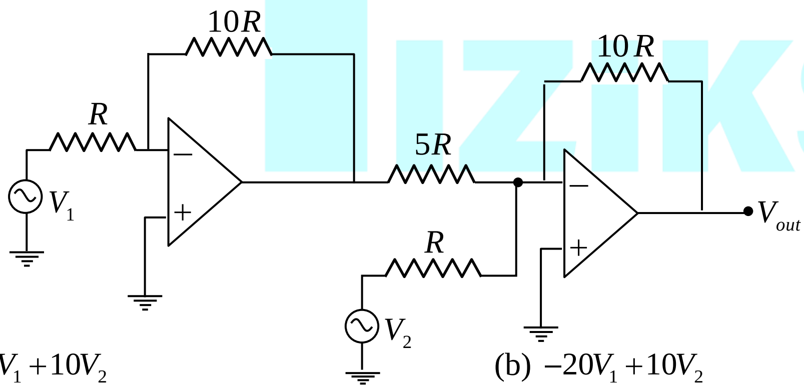
(d)



Ans. 29: (b)

Solution: In equilibrium fermi-level on each side aligned.

Q30. In the circuit shown in the figure, both OPAMPs are ideal. The output for the circuit V_{out} is



(a) $20V_1 + 10V_2$ (b) $-20V_1 + 10V_2$

(c) $10V_1 - 20V_2$ (d) $20V_1 - 10V_2$

Ans. 30: (d)

Solution: Output of first op-amp is $V_{01} = -\dfrac{10R}{R}V_1 = -10V_1$.

$$\text{Thus } V_{out} = -\frac{10R}{5R}(-10V_1) + \left(-\frac{10R}{R}\right)V_2 \Rightarrow V_{out} = 20V_1 - 10V_2$$

SECTION - B

MULTIPLE SELECT QUESTIONS (MSQ)

Q31 – Q40 carry two marks each.

Q31. If P and Q are Hermitian matrices which of the following is/are true?

(A matrix P is Hermitian if $P = P^\dagger$, where the elements $P_{ij}^\dagger = P_{ji}^*$)

(a) $PQ + QP$ is always Hermitian

(b) $i(PQ - QP)$ is always Hermitian

(c) PQ is always Hermitian

(d) $PQ - QP$ is always Hermitian

Ans. : (a), (b)

Solution: $(PQ+QP)^\dagger = (PQ)^\dagger + (QP)^\dagger = Q^\dagger P^\dagger + P^\dagger Q^\dagger = QP + PQ$

$(PQ)^\dagger = Q^\dagger P^\dagger = QP$

$(PQ - QP)^\dagger = QP - PQ$

$\left[i(PQ-QP)\right]^\dagger = -i\left(Q^\dagger P^\dagger - P^\dagger Q^\dagger\right) = -i(QP - PQ) = i(PQ - QP)$

Q32. Consider a vector function $\vec{u}(\vec{r})$ and two scalar functions $\psi(\vec{r})$ and $\phi(\vec{r})$. The unit vector $\hat{n}$ is normal to the elementary surface dS, dV is an infinitesimal volume, $\overrightarrow{dl}$ is an infinitesimal line element and $\frac{\partial}{\partial n}$ denotes the partial derivative along $\hat{n}$. Which of the following identities is/are correct?

(a) $\int_V \vec{\nabla}.\vec{u} dV = \oiint_S \vec{u}\cdot\hat{n} dS$, where surface S bounds the volume V

(b) $\int_V \left[\psi \nabla^2\phi - \phi\nabla^2\psi\right] dV = \oiint_S \left[\psi\frac{\partial\phi}{\partial n} - \phi\frac{\partial\psi}{\partial n}\right] dS$, where surface S bounds the volume V

(c) $\int_V \left[\psi \nabla^2\phi - \phi\nabla^2\psi\right] dV = \oiint_S \left[\psi\frac{\partial\phi}{\partial n} + \phi\frac{\partial\psi}{\partial n}\right] dS$, where surface S bounds the volume V

(d) $\oint_C \vec{u}\cdot\overrightarrow{dl} = \iint_S \left(\vec{\nabla}\times\vec{u}\right)\cdot\hat{n} dS$, where C is the boundary of surface S

Ans. 32: (a), (b) and (d)

Solution: (a) Gauss Divergence Theorem

(b) $\int_V \psi\nabla^2\phi dV = \int_V \psi\vec{\nabla}.\left(\vec{\nabla}\phi\right)dV = \oint_S \left(\vec{\nabla}\phi\right).d\vec{S} = \oint_S \psi\frac{\partial\phi}{\partial n}dS$

Similarly $-\int_V \phi\nabla^2\psi dV = -\int_V \phi\vec{\nabla}.\left(\vec{\nabla}\psi\right)dV = -\oint_S \phi\left(\vec{\nabla}\psi\right).d\vec{S} = -\oint_S \phi\frac{\partial\psi}{\partial n}dS$

Hence option (b) is correct.

(d) Stoke's Theorem

Q33. A thin rod of uniform density and length $2\sqrt{3}\,m$ is undergoing small oscillations about a pivot point. The time period of oscillation $\left(T_m\right)$ is minimum when the distance of the pivot point from the center-of-mass of the rod is x_m. Which of the following is/are correct?

(a) $x_m = 1m$ (b) $x_m = \frac{\sqrt{3}}{2}m$ (c) $T_m = \frac{2\pi}{\sqrt{3}}s$ (d) $T_m = \frac{2\pi}{\sqrt{5}}s$

Ans. : (a), (d)

Solution: Time period of rod of length L is $T = 2\pi\sqrt{\frac{I}{MgR}}$

where $I = \frac{Ml^2}{12} + Mx^2$, $R = x$ and $l = 2\sqrt{3}$

$$T^2 = 4\pi^2\left[\frac{Ml^2}{12\cdot Mgx} + \frac{Mx^2}{Mgx}\right] \Rightarrow T = 2\pi\sqrt{\frac{l^2}{12gx} + \frac{x}{g}}$$

$$\because T^2 = \frac{4\pi^2}{g}\left(\frac{l^2}{12x} + x\right)$$

$$\frac{dT^2}{dx} = 0 \Rightarrow \frac{4\pi^2}{g}\left(-\frac{l}{12x^2} + 1\right) = 0 \Rightarrow x^2 = \frac{l^2}{12} \Rightarrow x^2 = \frac{4\times 3}{12} = 1 \Rightarrow x = 1m$$

$$T = 2\pi\sqrt{\frac{l^2}{12gx} + \frac{x}{g}} = 2\pi\sqrt{\frac{1}{10} + \frac{1}{10}} = 2\pi\sqrt{\frac{2}{10}} = \frac{2\pi}{\sqrt{5}}\text{ sec.}$$

Hence option (a) and (d) are correct.

Q34. Three sinusoidal waves off the same frequency travel with the same speed along the positive x-direction. The amplitudes of the waves are a, $\frac{a}{2}$ and $\frac{a}{3}$ and the phase constants of the waves are $\frac{\pi}{2}$, π and $\frac{3\pi}{2}$, respectively. If A_m and φ_m are the amplitude and phase constant of the wave resulting from the superposition of the three waves, which of the following is/are correct?

(a) $A_m = \frac{5}{6}a$ (b) $\varphi_m = \frac{\pi}{2} + \tan^{-1}\left(\frac{3}{4}\right)$

(c) $A_m = \frac{7}{6}a$ (d) $\varphi_m = \tan^{-1}\left(\frac{2}{3}\right)$

Ans. : (a), (b)

Solution: $y_1 = a\sin\left(\omega t + \frac{\pi}{2}\right) = a\cos\omega t$, $y_2 = \frac{a}{2}\sin(\omega t + \pi) = -\frac{a}{2}\sin\omega t$,

$$y_3 = \frac{a}{3}\sin\left(\omega t + \frac{3\pi}{2}\right) = -\frac{a}{3}\cos\omega t$$

$$y = y_1 + y_2 + y_3 = \frac{2a}{3}\cos\omega t - \frac{a}{2}\sin\omega t = A_m \sin(\omega t + \theta)$$

$$A_m = \sqrt{\left(\frac{2a}{3}\right)^2 + \left(-\frac{a}{2}\right)^2} = a\sqrt{\frac{4}{9} + \frac{1}{4}} = a\sqrt{\frac{16+9}{36}} = \frac{5}{6}a$$

$$A_m \cos\phi_m = -\frac{a}{2}, \quad A_m \sin\phi_m = \frac{2a}{3}$$

$$\Rightarrow \tan\phi_m = -\frac{2a}{3}\frac{2}{a} = -\frac{4}{3} \Rightarrow -\tan\phi_m = \frac{4}{3} \Rightarrow -\cot\left(\frac{\pi}{2} - \phi_m\right) = \frac{4}{3} \Rightarrow \cot\left(\phi_m - \frac{\pi}{2}\right) = \frac{4}{3}$$

$$\Rightarrow \tan\left(\phi_m - \frac{\pi}{2}\right) = \frac{3}{4} \Rightarrow \phi_m - \frac{\pi}{2} = \tan^{-1}\frac{3}{4} \Rightarrow \phi_m = \frac{\pi}{2} + \tan^{-1}\left(\frac{3}{4}\right)$$

Q35. An object executes simple harmonic motion along the x-direction with angular frequency ω and amplitude a. The speed of the object is $4cm/s$ and $2cm/s$ when it is at distances $2cm$ and $6cm$ respectively from the equilibrium position. Which of the following is/are correct?

(a) $\omega = \sqrt{\frac{3}{8}}\text{ rad/s}$ (b) $\omega = \sqrt{\frac{5}{6}}\text{ rad/s}$ (c) $a = \sqrt{\frac{140}{3}}\text{ cm}$ (d) $a = \sqrt{\frac{175}{6}}\text{ cm}$

Ans. : (a), (c)

Solution: $x = A\sin(\omega t)$

$$\Rightarrow u = \frac{dx}{dt} = A\omega\cos(\omega t) = A\omega\sqrt{1 - \frac{x^2}{A^2}} = \omega\sqrt{A^2 - x^2} \Rightarrow u^2 = \omega^2\left(A^2 - x^2\right)$$

$\therefore (4)^2 = \omega^2 (a^2 - 2^2)$ (i)

$(2)^2 = \omega^2 (a^2 - 6^2)$ (ii)

Divide (i) and (ii), we get

$$\frac{16}{4} = \frac{a^2 - 4}{a^2 - 36} \Rightarrow a = \sqrt{\frac{140}{3}}$$

$$\therefore \omega = \frac{u}{\sqrt{a^2 - x^2}} = \frac{2}{\sqrt{\frac{140}{3} - 6^2}} = \frac{2}{\sqrt{\frac{140}{3} - 36}} = \sqrt{\frac{3}{8}} \text{ rad/sec}$$

Thus, $a = \sqrt{\frac{140}{3}}$ and $\omega = \sqrt{\frac{3}{8}}$ rad/sec

Q36. For electric and magnetic field $\vec{E}$ and $\vec{B}$, due to a charge density $\rho(\vec{r}, t)$ and a current density $\vec{J}(\vec{r}, t)$, which of the following relations is/are always correct?

(a) $\vec{\nabla} \times \vec{E} = 0$

(b) $\vec{\nabla} \cdot \vec{B} = 0$

(c) $\vec{\nabla} \cdot \vec{J} - \frac{\partial \rho}{\partial t} = 0$

(d) $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$, where $\vec{F}$ is the force on a particle with charge q moving with velocity $\vec{v}$

Ans. : (b), (d)

Solution:

Q37. A spherical dielectric shell with inner radius a and outer radius b, has polarization $\vec{P} = \frac{k}{r^2}\hat{r}$, where k is a constant and $\hat{r}$ is the unit vector along the radial direction. Which of the following statements is/are correct?

(a) The surface density of bound charges on the inner and outer surfaces are $-k$ and $+k$, respectively. The volume density of bound charges inside the dielectric is zero

(b) The surface density of bound charges is zero on both the inner and outer surfaces. The volume density of bound charges inside the dielectric is $+k$

(c) The surface density of bound charges on the inner and outer surfaces are $\frac{-k}{a^2}$ and $\frac{k}{b^2}$, respectively. The volume density of bound charges inside the dielectric is zero

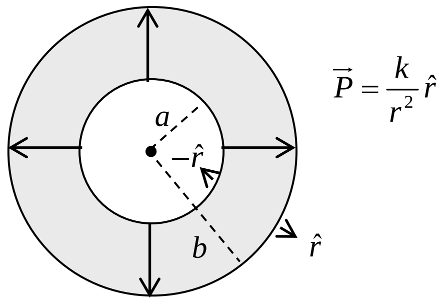
(d) The surface density of bound charges is zero on both the inner and outer surfaces. The volume density of bound charges inside the dielectric is $\frac{3k}{4\pi(b^3 - a^3)}$

Ans. 37: (c)

Solution: $\rho_b = -\vec{\nabla}.\vec{P} = -\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \times \frac{k}{r^2}\right) = 0$

$$\sigma_b\left(r=a\right) = \vec{P}.\hat{n} = \frac{k}{a^2}\hat{r}.\left(-\hat{r}\right) = -\frac{k}{a^2}$$

and $\sigma_b\left(r=b\right) = \vec{P}.\hat{n} = \frac{k}{b^2}\hat{r}.\left(\hat{r}\right) = \frac{k}{b^2}$



Q38. One mole of an ideal gas having specific heat ratio (γ) of 1.6 is mixed with one mole of another ideal gas having specific heat ratio of 1.4. If C_V and C_P are the molar specific heat capacities of the gas mixture at constant volume and pressure, respectively, which of the following is/are correct? (R denotes thee universal gas constant).

(a) $C_V = 2.08R$ (b) $C_p = 2.9R$ (c) $C_p = 1.48C_V$ (d) $C_p = 1.52C_V$

Ans. : (a), (c)

Solution: $n_1 = 1, n_2 = 1, \gamma_1 = 1.6, \gamma_2 = 1.4$, $C_V = \frac{R}{\gamma - 1}$

$$C_V = \frac{n_1 C_{V_1} + n_2 C_{V_2}}{n_1 + n_2} = \frac{R}{2}\left[\frac{1}{1.6-1} + \frac{1}{1.4-1}\right] = 2.08R$$

$$C_p = C_V + R = 3.08R \Rightarrow \frac{C_p}{C_V} = \frac{3.08}{2.08} = 1.48$$

Q39. Two relativistic particles with opposite velocities collide head-on and come to rest by sticking with each other. Which of the following quantities is/are conserved in the collision?

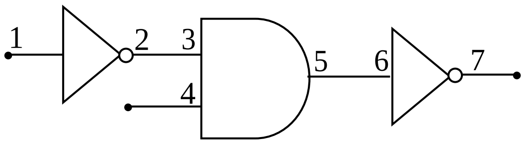
(a) Total momentum (b) Total energy

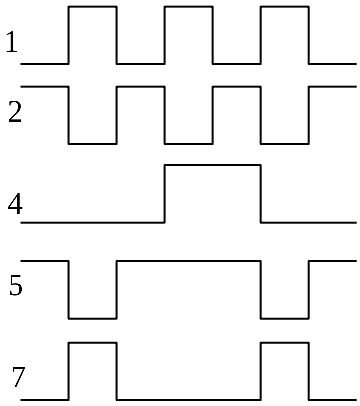
(c) Total kinetic energy (d) Total rest mass

Ans. : (a), (b)

Solution:

Q40. Figure shows a circuit diagram comprising Boolean logic gates and the corresponding timing diagrams show the digital signals at various points in the circuit. Which of the following is/are true?





(a) Points 3 and 7 are shorted

(b) The NOT gate on the right is faulty

(c) The AND gate is faulty and acts like a NOR gate

(d) The AND gate is faulty and acts like an OR gate

Ans. : (d)

Solution:

SECTION - C

NUMERICAL ANSWER TYPE (NAT)

Q41 - Q50 carry one mark each.

Q41. The line integral of the vector function $\vec{u}(x,y)=2y\,\hat{i}+x\,\hat{j}$ along the straight line from $(0,0)$ to $(2,4)$ is________

Ans. 41: 12

Solution: $\vec{u}=2y\hat{i}+x\hat{j}$, $(0,0)\rightarrow(2,4)$, Line equation is $y=2x\Rightarrow dy=2dx$ and $d\vec{l}=dx\hat{i}+dy\hat{j}$.

$$\Rightarrow \vec{u}.d\vec{l}=2ydx+xdy\Rightarrow \vec{u}.d\vec{l}=2\times 2x\times dx+x(2dx)\Rightarrow \vec{u}.d\vec{l}=6xdx$$

Thus $\int \vec{u}.d\vec{l}=\int_0^2 6xdx=6\left[\frac{x^2}{2}\right]_0^2=12$

Q42. Consider a thin bi-convex lens of relative refractive index $n=1.5$. The radius of curvature of one surface of the lens is twice that of the other. The magnitude of larger radius of curvature in units of the focal length of the lens is ________ (Round off to 1 decimal place)

Ans. : 1.5

Solution: $\frac{1}{f}=(\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)\Rightarrow\frac{1}{f}=(1.5-1)\left(\frac{1}{R}-\frac{1}{-2R}\right)=0.5\left(\frac{1}{R}+\frac{1}{2R}\right)$

$$\Rightarrow\frac{1}{f}=\frac{0.5\times 3}{2R}\Rightarrow 4R=3f\quad\Rightarrow 2R=\frac{3f}{2}=1.5f$$

Q43. Water flows in a horizontal pipe in a streamlined manner at an absolute pressure of $4\times10^5\,Pa$ and speed of $6\,m/s$. If it exits the pipe at a pressure of $10^5\,Pa$, the speed of water at the exit point is _________ m/s (Round off to 1 decimal place)

(The density of water is $1000\,kg/m^3$)

Ans. : 25.2

Solution: Applying Bernoulli's equation

$$P_1+\frac{1}{2}\rho V_1^2+\rho gh_1=P_2+\frac{1}{2}\rho V_2^2+\rho gh_2$$

As $h_1=h_2\Rightarrow P_1+\frac{\rho V_1^2}{2}=P_2+\frac{\rho V_2^2}{2}$

$$4\times10^5+1000\times\frac{6\times6}{2}=10^5+1000\times\frac{V_2^2}{2}$$

$$\Rightarrow V_2=25.21\,m/\sec$$

Q44. Consider a retarder with refractive indices $n_e = 1.551$ and $n_o = 1.542$ along the extraordinary and ordinary axes, respectively. The thickness of this retarder for which a left circularly polarized light of wavelength $600\,nm$ will be converted into a right circularly polarized light is ______ μm. (Round off to 2 decimal place)

Ans. : 33.33

Solution: $(n_e - n_o)t = \frac{\lambda}{2} \Rightarrow t = \frac{\lambda}{2(n_e - n_o)} = \frac{600\times10^{-9}}{2(1.551-1.542)}m$

$$\Rightarrow t = \frac{600}{2\times0.009}\times10^{-9} = \frac{600}{2\times9}\times10^{-6} = \frac{100}{3}\times10^{-6} = 33.33\times10^{-6}\,m = 33.33\,\mu m$$

Q45. Using a battery a $10\,pF$ capacitor is charged to $50V$ and then the battery is removed. After that, a second uncharged capacitor is connected to the first capacitor in parallel. If the final voltage across the second capacitor is $20V$, its capacitance is _________ pF.

Ans. 45: 15

Solution: Case I: Capacitor is charged upto $q = CV = 10\,pF \times 50V = 500\,pC$,

Case II: First Capacitor is connected in parallel to second and common voltage is 20 V.

$q = q_1 + q_2 = 500\,pC \Rightarrow 10\,pF \times 20V + q_2 = 500\,pC$

$\Rightarrow q_2 = 300\,pC \Rightarrow C_2 \times 20V = 300\,pC \Rightarrow C_2 = 15\,pF$

Q46. Consider two spherical perfect blackbodies with radii R_1 and R_2 at temperatures $T_1 = 1000K$ and $T_2 = 2000\,K$, respectively. They both emit radiation of power $1kW$. The ratio of their radii R_1 / R_2 is given by__________

Ans. : 4

Solution: $P_1 = \sigma 4\pi R_1^2 T_1^4$ and $P_2 = \sigma 4\pi R_2^2 T_2^4$

$$\because \frac{P_1}{P_2} = 1 \Rightarrow R_1^2 T_1^4 = R_2^2 T_2^4 \Rightarrow \frac{R_1^2}{R_2^2} = \frac{T_2^4}{T_1^4} \Rightarrow \frac{R_1}{R_2} = \frac{T_2^2}{T_1^2} = \frac{(2000)^2}{(1000)^2} = 4$$

Q47. In a Compton scattering experiment, the wavelength of incident X-rays is $0.500\,\overset{0}{\text{A}}$. If the Compton wavelength λ_c is $0.024\,\overset{0}{\text{A}}$, the value of the longest wavelength possible for the scattered X-ray is _________ $\overset{0}{\text{A}}$ (specify up to 3 decimal places)

Ans. : 0.548

Solution: $\Delta\lambda = \frac{h}{mc}(1-\cos\phi)$

$\lambda_2 - \lambda_1 = \frac{h}{mc}\times 2$, $\lambda_2 = 0.024\times2 + 0.500 = 0.048 + 0.500 = 0.548\,\overset{0}{\text{A}}$

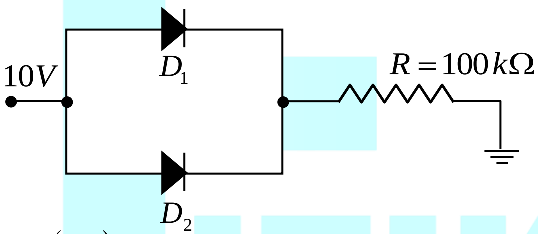
Q48. A solid with FCC crystal structure is probed using X-rays of wavelength $0.2\,nm$. For the crystallographic plane given by $(2,0,0)$, a first order diffraction peak is observed for a Bragg angle of 21^0. The unit cell size is __________ nm (Round off to 2 decimal places)

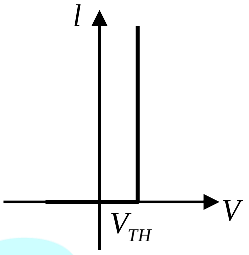
Ans. : 0.56

Solution: $2d\sin\theta = \lambda \Rightarrow \dfrac{2a\sin\theta}{\sqrt{h^2+k^2+l^2}} = \lambda$

$$\Rightarrow a = \frac{\lambda}{2\sin\theta}\sqrt{h^2+k^2+l^2} = \frac{0.2\times10^{-9}}{2\times\sin\left(21^0\right)}\sqrt{2^2+0+0} = \frac{0.2nm}{2\times0.36}\times2 = 0.56\,nm$$

Q49. The figure shows a circuit containing two diodes D_1 and D_2 with threshold voltages V_{TH} of $0.7V$ and $0.3V$, respectively. Considering the simplified diode model, which assumes diode I-V characteristic as shown in the plot on the right, the current through the resistor R is ________ μA.



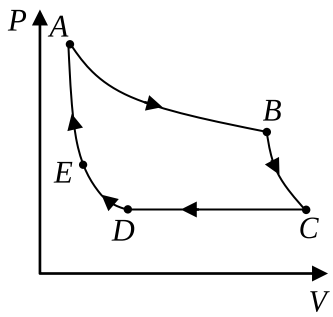


Ans. 49: 97

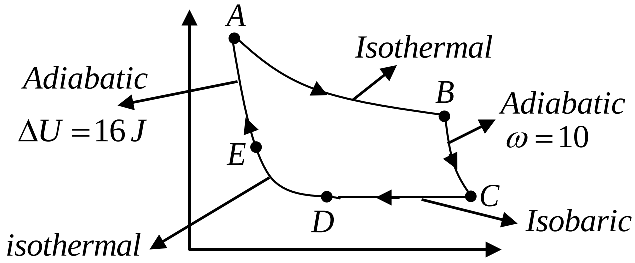
Solution: Diodes $D_1(Si)$ and $D_2(Ge)$ are in parallel. So once switch is ON, $D_2(Ge)$ will be ON and $D_1(Si)$ will be OFF.

$$I = \frac{10V - 0.3V}{100\,k\Omega} = \frac{9.7}{100}mA = 97\,\mu A$$

Q50. An ideal gas undergoes an isothermal expansion along a path AB, adiabatic expansion along BC, isobaric compression along CD, isothermal compression along DE, and adiabatic compression along EA, as shown in the figure. The work done by the gas along the process BC is $10\,J$. The change in the internal energy along process EA is $16\,J$. The absolute value of the change in the internal energy along the process CD is ________ J



Ans. : 6



Solution: $\Delta U_{AB} = 0$, $\Delta U_{DE} = 0$

$\because \Delta U = 0$

$\Delta U_{BC} + \Delta U_{CD} + \Delta U_{EA} = 0$

$-10 + \Delta U_{CD} + 16 = 0 \quad \Rightarrow \Delta U_{CD} = -6$

Q51 - Q60 carry two marks each.

Q51. If a function $y(x)$ is described by the initial-value problem $\frac{d^2y}{dx^2}+5\frac{dy}{dx}+6y=0$, with initial conditions $y(0)=2$ and $\left(\frac{dy}{dx}\right)_{x=0}=0$, then the value of y at $x=1$ is _________

(Round off to 2 decimal places)

Ans. : 0.60

Solution: $y=c_1e^{-2x}+c_2e^{-3x}$ $\qquad \because y(0)=2$

$\Rightarrow 2=c_1+c_2$ (i)

$y'=-2c_1e^{-2x}-3c_2e^{-3x}$ $\qquad \because \left(\frac{dy}{dx}\right)_{x=0}=0$

$\Rightarrow 0=-2c_1-3c_2$ (ii)

Solve (i) and (ii) we will get; $c_1=6$, $c_2=-4$

$\Rightarrow y=6e^{-2x}-4e^{-3x} \Rightarrow y(1)=0.6$

Q52. A vehicle of mass $600\,kg$ with an engine operating at constant power P accelerates from rest on a straight horizontal road. The vehicle covers a distance of $600\,m$ in 1 minute. Neglecting all losses, the magnitude of P is _______ kW. (Round off to 2 decimal places)

Ans. : 1.11 to 1.13

Solution:

Q53. The angular momentum of a particle relative to origin varies with time (t) as $\vec{L}=\left(4\hat{x}+at^2\hat{y}\right)kgm^2/s$, where $a=1kg\,m^2/s^3$. The angle between $\vec{L}$ and the torque acting on the particle becomes 45^0 after a time of _________ s.

Ans. : 2

Solution: $\vec{L}=\left(4i+\alpha t^2\hat{j}\right)$

$$\vec{\tau}=\frac{d\vec{L}}{dt}=2\alpha t\hat{j}$$

$$\cos\left(\frac{\pi}{4}\right)=\frac{\vec{L}\cdot\vec{\tau}}{|L||\tau|}$$

$$\frac{1}{\sqrt{2}}=\frac{2\alpha^2t^3}{\sqrt{16+\alpha^2t^4}\times 2\alpha t}=\frac{\alpha t^2}{\sqrt{16+\alpha^2t^4}}$$

$16+\alpha^2t^4=2\alpha^2t^4 \Rightarrow \alpha^2t^4=16$

$$t^4 = \frac{16}{\alpha^2} = \frac{16}{1}$$

$$t = (16)^{1/4} = 2\sec$$

Q54. Two transverse waves $y_1 = 5\cos(kx - \omega t)$ cm, and $y_2 = 5\cos(kx + \omega t)$ cm, travel on a string along x-axis. If the speed of a point at $x = 0$ is zero at $t = 0s, 0.25s$ and $0.5s$, then the minimum frequency of the waves is __________Hz.

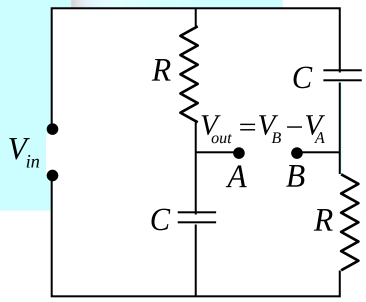
Ans. : 2

Solution: Time difference (Separation) between two zero velocities

$$\frac{T}{2} = 0.25s, \quad T = 0.5s,$$

So, frequency $f = \frac{1}{T} = \frac{1}{0.5} = 2\ Hz$

Q55. For the ac circuit shown in the figure, $R = 100k\Omega$ and $C = 10pF$, the phase difference between V_{in} and V_{out} is 90^0 at the input signal frequency of ________ kHz. (Round off to 2 decimal places)



Ans. 55: 159.2

Solution: $V_A = \left(\frac{X_C}{R + X_C}\right)V_{in}$ and $V_B = \left(\frac{R}{R + X_C}\right)V_{in}$

$$\Rightarrow V_{out} = V_B - V_A \Rightarrow V_{out} = \left(\frac{R - X_C}{R + X_C}\right)V_{in} \Rightarrow \frac{V_{out}}{V_{in}} = \left(\frac{j\omega CR - 1}{j\omega CR + 1}\right)$$

$$\Rightarrow \frac{V_{out}}{V_{in}} = \left(\frac{\sqrt{1 + (\omega CR)^2}}{\sqrt{1 + (\omega CR)^2}}\right)\frac{e^{-j\theta}}{e^{j\theta}} = e^{-j2\theta} \qquad \text{where } \theta = \tan^{-1}(\omega RC)$$

Thus $\phi = -2\theta = -2\tan^{-1}(\omega RC) \Rightarrow -\frac{\pi}{2} = -2\tan^{-1}(2\pi fRC) \Rightarrow 2\pi fRC = 1$

$$\Rightarrow f = \frac{1}{2\pi RC} \Rightarrow f = \frac{1}{2 \times 3.14 \times (100 \times 10^3\Omega)(10 \times 10^{-12}F)} Hz$$

$$\Rightarrow f = \frac{10^6}{2 \times 3.14} Hz = \frac{1000}{2 \times 3.14} kHz = 159.2\, kHz$$

Q56. The magnetic fields in tesla in the two regions separated by the $z=0$ plane are given by $\vec{B}_1 = 3\hat{x}+5\hat{z}$ and $\vec{B}_2 = \hat{x}+3\hat{y}+5\hat{z}$. The magnitude of the surface current density at the interface between the two regions is $\alpha\times10^6\,A/m$. Given the permeability of the free space $\mu_0 = 4\pi\times10^{-7}\,N/A^2$, the value of α is ________. (Round off 2 decimal places)

Ans. 56: 2.86

Solution: $\vec{B}_1 = 3\hat{x}$ and $\vec{B}_2 = \hat{x}+3\hat{y}$. Thus $\vec{B}_1 - \vec{B}_2 = 2\hat{x}-3\hat{y} \Rightarrow \left|\vec{B}_1 - \vec{B}_2\right| = \sqrt{4+9} = \sqrt{13}$

$$\because \left|\vec{B}_1 - \vec{B}_2\right| = \mu_0 K \Rightarrow \mu_0 K = \sqrt{13} \Rightarrow K = \frac{\sqrt{13}}{\mu_0} = \frac{3.6}{4\pi\times10^{-7}} = 2.86\times10^6$$

$\Rightarrow \alpha = 2.86$

Q57. A body at a temperature T is brought into contact with a reservoir at temperature $2T$. Thermal equilibrium is established at constant pressure. The heat capacity of the body at constant pressure is C_P. The total change in entropy of the body and the reservoir in units of C_p is ________. (Round off 2 decimal places)

Ans. : 0.19 to 0.20

Solution: $\Delta S_B = \int_T^{2T} C_P \frac{dT}{T} = C_P \ln 2$ Body heat absorbed positive

$\Delta S_R = \frac{1}{2T}\int_T^{2T} C_P dT = \frac{C_P T}{2T} = \frac{C_P}{2}$ Reservoir liberated negative

$\Delta S = C_P\left[0.693-0.5\right] = C_P 0.193$

Q58. One mole of an ideal monoatomic gas at pressure P, volume V and temperature T is expanded isothermally to volume $4V$. Thereafter, the gas is heated isochorically (at constant volume) till its pressure becomes P. If R is the universal gas constant, the total heat transfer in the process, in units of RT is ________. (Round off 2 decimal places)

Ans. : 5.88 to 5.94

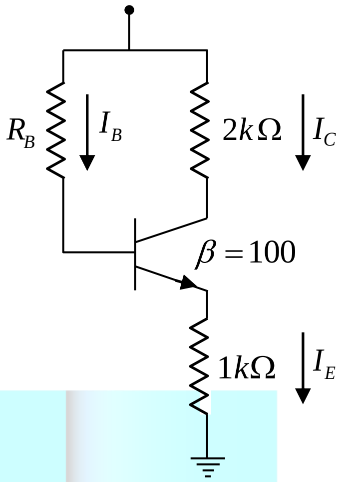
Solution: $dQ_1 = RT\ln\frac{V_2}{V_1} = RT\ln 4$ $dU = 0$ (isothermally),

$C_V = 1.5R$ (Monoatomic Gas)

$$dQ_2 = C_V dT = C_V\left(4T - T\right) = 3C_V T = \frac{9}{2}RT = 4.5RT$$

Total $dQ = RT\left(2\ln 2 + 4.5\right) = 5.886$

Q59. In the transistor circuit given in the figure, the emitter-base junction has a voltage drop of $0.7V \cdot A$ collector-emitter voltage of $14V$ reverse biases the collector. Assuming the collector current to be the same as the emitter current, the value of R_B is _________ $k\Omega$.



Ans. 59: 865

Solution: $V_{BE} = 0.7V$, $V_{CE} = 14V$ and $I_E \approx I_C$

$$\because V_{CE} = V_{CC} - I_C\left(R_C + R_E\right) \Rightarrow I_C = \frac{V_{CC} - V_{CE}}{R_C + R_E} = \frac{20V - 14V}{2k\Omega + 1k\Omega} = 2\,mA \approx I_E$$

Apply KVL in input section

$$-V_{CC} + \frac{I_C}{\beta}R_B + V_{BE} + I_E R_E = 0 \Rightarrow -20V + \frac{2\,mA}{100} \times R_B + 0.7V + 2\,mA \times 1k\Omega = 0$$

$$\Rightarrow \frac{2\,mA}{100} \times R_B = 17.3V \Rightarrow R_B = 865\,k\Omega$$

Q60. The radioactive nuclei ${}^{40}K$ decay to ${}^{40}Ar$ with a half-life of 1.25×10^9 years. The $\frac{{}^{40}K}{{}^{40}Ar}$ isotopic ratio for a particular rock is found to be 50. The age of the rock is $m \times 10^7$ years. The value of m is _________. (Round off to 2 decimal places)

Ans. : 3.57

Solution: $$\frac{K^{40}}{A^{40}} = \frac{N_0 e^{-\lambda t}}{N_0 - N_0 e^{-\lambda t}} = 50 \Rightarrow e^{\lambda t} = \frac{51}{50} = 1.02 \Rightarrow \lambda t = \ln(1.02)$$

$$\Rightarrow t = \frac{1.25 \times 10^9}{0.693}\ln(1.02) \Rightarrow t = 3.57 \times 10^7 \text{ years.}$$